

Estimating σ -meson couplings from $D \rightarrow 3\pi$ decays

Claudio Dib ⁽¹⁾ and Rogerio Rosenfeld ⁽²⁾

(1) Universidad Técnica Federico Santa María

Departamento de Física

1049-001 Valparaíso, Chile

E-mail address: cdib@fis.utfsm.cl

(2) Instituto de Física Teórica

R. Pamplona 145, 01405-900 São Paulo - SP, Brazil

E-mail address: rosenfel@ift.unesp.br

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Using recent experimental evidence from E791 on the sigma meson in $D \rightarrow 3\pi$ decays, we study the relevant couplings in $D \rightarrow \sigma\pi$ and $\sigma \rightarrow \pi\pi$ within the accepted theoretical framework for non leptonic D decays, finding an overall consistency of the theory with the experimental data.

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I. INTRODUCTION

We know that QCD is the right theory for the strong interactions at high energies [1] and also it is well accepted as the fundamental underlying theory at low energies. However, due to infrared slavery, it is very difficult to apply this theory in the hadronic realm because the resulting large coupling constant at those lower energies makes it impossible to use perturbation theory. Besides *ab initio* calculations, like lattice techniques, one has to resort to an effective theory (chiral perturbation theory, ChPT [2]) or sensible models which describe the relevant degrees of freedom restricted by the symmetries of the low energy domain. An example of the latter is the sigma model, [3] which correctly describes $\pi\pi$ scattering near threshold, to leading order in a momentum expansion (and which coincides with ChPT at that order). However, when one increases the center-of-mass energy, the cross section starts to increase until resonances are reached. There is a long standing controversy in the literature about the existence of an isospin zero, broad scalar resonance in $\pi\pi$ scattering, the so-called sigma meson resonance [4]. Recently, a whole conference was devoted to the study of the possible existence of the sigma resonance [5]. The controversy basically stems from the large width of the resonance, which makes it difficult to discern if the shape of the spectrum is actually due to a pole in the amplitude or to a result of other effects in the s and t channels. Several experimental results can be explained with the existence of such a resonance [6]. If it exists, it is a relevant degree of freedom and must be incorporated into the analysis, together with the conditions imposed by unitarity, chiral symmetry, etc [7]. It also must show up in systems other than the $\pi\pi$ system. For instance, it was pointed out ten years ago that the sigma meson can have an important role in explaining the $\Delta I = 1/2$ enhancement in $K \rightarrow \pi\pi$ decays [8]. Recently it has been found strong experimental evidence that the sigma meson is very important in the D-meson system, in the singly Cabibbo-suppressed decay $D^+ \rightarrow \pi^+\pi^+\pi^-$, being responsible for approximately half of the decays through the resonant sequence: $D^+ \rightarrow \pi^+\sigma \rightarrow \pi^+\pi^+\pi^-$ [9]. A best fit to the Dalitz plot of this decay results in $m_\sigma = 483 \pm 31$ MeV and $\Gamma_\sigma = 338 \pm 48$ MeV,

where statistical and systematic errors have been added in quadrature. We want to explore the consequences of this experimental result in the context of the well known theories of light mesons and weak decays. While we were preparing this work, another study appeared on the effect of the σ in different weak processes [10]. They conclude that the new result of E791, with a smaller value of the non-resonant contribution to $\Gamma(D \rightarrow 3\pi)$ is in better agreement with the well measured $\Gamma(D \rightarrow 2\pi)$, to which they relate via PCAC. One should notice however, that PCAC must be used with care, since it is valid only when the pion is soft, and not in the entire kinematic range of these decays. Recently, the $D \rightarrow \sigma\pi$ process was also studied in the context of a constituent quark-meson model [11]. Here we study the specific consequences of a sigma particle and its couplings related to the $D \rightarrow 3\pi$ process, invoking two formulations: chiral symmetry as expressed in the linear sigma model to relate the $\sigma\pi\pi$ coupling to other observables and the QCD-inspired phenomenological model of Bauer, Stech and Wirbel [12] to treat the non-leptonic D decay.

This paper is organized as follows. In section II we extract the effective couplings involved in this D decay from the experimental data. In section III we explore the consistency of this sigma meson within the models for spontaneous chiral symmetry breaking of strong interactions. In section IV we address the problem of estimating the weak process $D \rightarrow \sigma\pi$ in the standard model of weak interactions and within the treatment of Bauer, Stech and Wirbel for D decays. Our conclusions are presented in section V.

II. THE COUPLINGS IN THE DECAY $D \rightarrow \sigma\pi \rightarrow 3\pi$

From the experimental result that a fraction $f = (44 \pm 10)\%$ of the $D^+ \rightarrow \pi^+\pi^+\pi^-$ goes through the resonant channel [9], one can estimate the amplitude of the weak process $D^+ \rightarrow \pi^+\sigma$. By resonant decay in this process one understands $D \rightarrow \sigma\pi^+ \rightarrow \pi^+\pi^-\pi^+$, *i.e.* a $\pi^+\pi^-$ pair in the final state appears through the formation of an intermediate σ (sigma) resonance. The coupling $D - \sigma - \pi$ in this resonant process is, in general, a function of the

q^2 of the virtual sigma (*i.e.* the invariant mass of the $\pi^+\pi^-$ pair). This issue would not be a problem if the sigma were narrow, because then the amplitude would be strongly dominated at $q^2 = m_\sigma^2$, fixing the couplings to a constant value on the mass shell of the intermediate particle. As a leading approximation, we assume that the coupling is not a strongly varying function of q^2 and fix it on the mass shell of the sigma. A more precise estimate would require a model for the D decay into three pions via a sigma. Assuming constant couplings $g_{D\sigma\pi}$ and $g_{\sigma\pi\pi}$ we have for the resonant three-body decay width:

$$\Gamma(D^+ \rightarrow \sigma\pi^+ \rightarrow \pi^+\pi^-\pi^+) = \frac{1}{2} \frac{1}{2m_D} g_{D\sigma\pi}^2 g_{\sigma\pi\pi}^2 \int_{4m_\pi^2}^{(m_D-m_\pi)^2} \frac{dq^2}{2\pi} \frac{1}{8\pi} \lambda^{1/2} \left(1, \frac{q^2}{m_D^2}, \frac{m_\pi^2}{m_D^2}\right) \quad (1)$$

$$\times \frac{1}{8\pi} \lambda^{1/2} \left(1, \frac{m_\pi^2}{q^2}, \frac{m_\pi^2}{q^2}\right) \frac{1}{(q^2-m_\sigma^2)^2 + \Gamma_\sigma(q^2)^2 m_\sigma^2},$$

where we use the standard notation $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and where each factor $1/(8\pi) \times \lambda^{1/2}$ is the phase space integral of the corresponding two-body decay subprocess. The factor $\frac{1}{2}$ in front is due to the symmetry of the two π^+ in the final state, which the integral consider as distinguishable. Also $\Gamma_\sigma(q^2) \equiv \Gamma_\sigma^0 \times (m_\sigma/q) (p^*(q^2)/p^*(m_\sigma^2))$ is the co-moving resonance width; here $p^*(q^2) = \sqrt{q^2/4 - m_\pi^2}$. A value for the strong coupling $g_{\sigma\pi\pi}$ can be obtained from the sigma width Γ_σ^0 by considering that the σ meson decays 100% into $\pi\pi$, two thirds of the time into charged pions. Using the general expression for the decay of a J=0 into two J=0 particles in terms of the coupling constant [13], we have

$$\frac{2}{3}\Gamma_\sigma^0 = \Gamma(\sigma \rightarrow \pi^+\pi^-) = g_{\sigma\pi\pi}^2 \frac{1}{8\pi m_\sigma^2} p^*, \quad (2)$$

where $p^* = \frac{1}{2}\lambda^{1/2}(m_\sigma^2, m_\pi^2, m_\pi^2)$ is the magnitude of the 3-momentum of either of the final particles in the CM frame.

For our numerical results we will use the data $m_D = 1869$ MeV, $m_\pi^+ = 139.6$ MeV [14] and the experimentally extracted values of the σ resonance (mass and width) $m_\sigma = 483 \pm 31$ MeV and $\Gamma_\sigma^0 = (338 \pm 48)$ MeV [9]. The value for the $g_{\sigma\pi^+\pi^-}$ coupling thus found is:

$$g_{\sigma\pi^+\pi^-} = (2.59 \pm 0.19) \text{ GeV}. \quad (3)$$

This result, together with the numerical value of the integral shown in Eq. 1, which is $(2.55 \pm 0.55) \times 10^{-3} \text{ GeV}^{-2}$ for all the appropriate values of the parameters, allows us to get an estimate for the weak $D - \sigma - \pi$ coupling:

$$g_{D\sigma\pi} = 654 \pm 120 \text{ eV}. \quad (4)$$

We have thus extracted this coupling using the same prescription for the resonance that gave the experimental values of the sigma mass and width cited above, and also assuming that the coupling is independent of the virtuality of the intermediate σ state. The validity of our estimate is thus conditioned to the assumption that the coupling for q^2 values away from the resonant peak does not differ by much from its value at the peak and that the shape of the resonance away from the peak is as prescribed in Eq. 1. Both approximations are clearly related and constitute the old issue of the background underlying the resonance.

To make a more crude estimate that does not deal with the background issue, one could use a narrow width approximation to estimate the coupling. Although this is *a priori* not a very good approximation since the σ resonance is not narrow, the estimate is rather robust because it hides the width of the resonance and only deals explicitly with its branching ratio. Within this approximation the resonant D decay width is:

$$\Gamma(D^+ \rightarrow \pi^+\pi^+\pi^-)_{res} \approx \frac{1}{2} \times \Gamma(D^+ \rightarrow \sigma\pi^+) \times Br(\sigma \rightarrow \pi^+\pi^-), \quad (5)$$

where again there is a symmetry factor $1/2$ due to the two identical π^+ . Now, $Br(\sigma \rightarrow \pi^+\pi^-) = 2/3$ and $\Gamma(D^+ \rightarrow \pi^+\pi^+\pi^-)_{res} = f \times \Gamma(D^+ \rightarrow \pi^+\pi^+\pi^-)$, according to the experimental analysis. Using the established data $Br(D^+ \rightarrow \pi^+\pi^-\pi^+) = (3.6 \pm 0.4) \times 10^{-3}$ and $\tau_{D^+} = (1.06 \pm 0.02) \times 10^{-12} \text{ s}$ [14], one thus obtains the estimate for the decay $D^+ \rightarrow \sigma\pi^+$ in this approximation:

$$\Gamma(D^+ \rightarrow \sigma\pi^+) = 3 \times f \times \Gamma(D^+ \rightarrow \pi^+\pi^-\pi^+) = (2.94 \pm 0.75) \times 10^{-12} \text{ MeV}. \quad (6)$$

From this rate we can again extract the $g_{D\sigma\pi}$ coupling using the general expression for the decay of a $J=0$ into two $J=0$ particles:

$$g_{D\sigma\pi}^2 = \frac{8\pi m_D^2}{p^*} \Gamma(D^+ \rightarrow \sigma\pi^+), \quad (7)$$

where $p^* = 1/2 \times \lambda^{1/2}(m_D^2, m_\sigma^2, m_\pi^2)$ is again the 3-momentum of either of the final particles in the CM frame. We thus obtain

$$g_{D\sigma\pi} \approx 548 \pm 70 \text{ eV}. \quad (8)$$

One can notice that the central value of this estimate is only 16% smaller than our more elaborate estimate of Eq. 4. In this sense, however crude the narrow width approximation could be considered *a priori*, it is indeed quite robust, since the result for $g_{D\sigma\pi}$ thus obtained does not differ much from our more elaborate estimate, in which the full resonance shape is taken into account.

III. THE σ MESON IN THE LINEAR SIGMA MODEL

Having estimated the couplings directly from the experimental data, let us see the consequences of these results within the theory. First, consider the $\sigma\pi\pi$ coupling. We are here in the low energy regime of strong interactions. Consider then the linear sigma model as the framework, with the σ meson as the scalar ($J^{PC} = 0^{++}$) that remains from the breakdown of chiral symmetry. It is sufficient to consider QCD with the lightest quarks u and d only, so that the chiral symmetry in question is $SU(2)_L \times SU(2)_R$ broken down to $SU(2)_V$ or isospin. The meson sector comprises the massive σ scalar and the three pions (pseudoscalars) π^a , $a = 1, 2, 3$, which play the role of Goldstone bosons. Using the notation of ref. [15] the quartic coupling of the potential, λ , and the vacuum expectation value of the field, v , determine all the physical parameters, like the mass of the sigma as $m_\sigma^2 = 2\lambda v^2$, the $\sigma\pi\pi$ coupling as $g_{\sigma\pi\pi} = 2\lambda v$ and the (charged) pion decay constant as $f_\pi = \sqrt{2}v$. The linear sigma model thus predicts the $g_{\sigma\pi\pi}$ coupling in terms of the mass of the σ particle and the pion decay constant:

$$g_{\sigma\pi\pi} = \frac{\sqrt{2}m_\sigma^2}{f_\pi} = (2.54 \pm 0.01) \text{ GeV}, \quad (9)$$

which is surprisingly close to the value deduced from the data on resonant D decay. Therefore, there is an apparent consistency of the linear sigma model description for the $\pi\pi$ resonance in D decays. However, a word of caution is due here: the validity of a perturbative calculation within a linear sigma model treated as a quantum theory is questionable if the mass of the scalar σ is large (*i.e.* if $m_\sigma \sim f_\pi$), which is precisely the case in the real world. A perturbative calculation could be trusted if the expansion parameter is much smaller than unity. In the sigma model, the expansion parameter is λ , or more precisely $6\lambda/(16\pi^2)$. From the data on m_σ and f_π it is easy to see that the value for the expansion parameter is:

$$\frac{6\lambda}{16\pi^2} \approx 0.5, \quad (10)$$

which is uncomfortably close to unity. This is precisely the reason why one would prefer to use a non-linear version of the model as a perturbative theory, where the heavy sigma is integrated out and only the light particles (the pions) are considered. The effective interaction among the pions is then explicitly weak at low momenta, vanishing at threshold (a feature which is evidence that chiral symmetry is spontaneously broken at low energies and that the pions are the corresponding Goldstone bosons). Here we used the linear sigma model with the sole purpose of exhibiting the sigma particle in the theory.

IV. THE $D^+ \rightarrow \sigma\pi^+$ FORM FACTOR IN THE BSW MODEL

The coupling $g_{D\sigma\pi}$ found in Eq. 4, which has dimensions of mass, is an effective result due to an underlying weak interaction process. In this section we want to examine this coupling at that fundamental level. We therefore estimate the relevant matrix elements of weak interactions that contribute the $D^+ \rightarrow \sigma\pi^+$ decay. However, because of the effect of strong interactions, the asymptotic states are not the elementary quarks but composite hadrons,

meaning that the analysis cannot be done purely at the level of fundamental interactions treated perturbatively. Only the weak interaction can be considered at leading order, but to take into account strong interactions we require of a sensible model. We thus study the problem within the well accepted model of Bauer, Stech and Wirbel (BSW) for non-leptonic D decays [12]. For our purposes, the σ particle will be a $J^{PC} = 0^{++}$ isoscalar with quark content $(\bar{u}u + \bar{d}d)$. In order to tackle non-leptonic decays, one has to use the operator product expansion to construct an effective weak hamiltonian containing local four-quark operators with Wilson coefficients $c_i(\mu)$ that can be computed perturbatively at the appropriate energy scale μ . In our case, the relevant effective hamiltonian is:

$$H_w = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} \left\{ c_1(\mu) (\bar{d}c)_{(V-A)} (\bar{u}d)_{(V-A)} + c_2(\mu) (\bar{u}c)_{(V-A)} (\bar{d}d)_{(V-A)} \right\}, \quad (11)$$

where $(\bar{q}q')_{(V-A)} = \bar{q}\gamma_\mu(1 - \gamma_5)q'$. In the BSW approach, the effective weak hamiltonian density is built in terms of products of hadron currents which mimic the underlying quark currents of the weak interactions:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} \left\{ a_1 J_\mu^{(\bar{d}c)} J^{\mu(\bar{u}d)} + a_2 J_\mu^{(\bar{u}c)} J^{\mu(\bar{d}d)} \right\}. \quad (12)$$

The coefficients a_1 and a_2 that accompany the products of currents are now phenomenological constants of the model that must be fitted from experiment. In this approach, the decay amplitude factorizes into a product of two current matrix elements. We will use this effective hamiltonian to study the $D^+ \rightarrow \sigma\pi^+$ decay process.

The factor proportional to a_1 in the matrix element $\langle \sigma\pi^+ | H_{eff} | D^+ \rangle$ involves two terms. One of them is $\langle \sigma\pi^+ | J_A^{\mu(\bar{u}d)} | 0 \rangle \langle 0 | J_{A,\mu}^{(\bar{d}c)} | D^+ \rangle$, so-called “annihilation” term, which we neglect within the model as it is prescribed, because it involves form factors of the light mesons at high momentum ($q^2 = m_D^2$) [12]. We should emphasize here that we are not neglecting the so-called *quark annihilation diagrams* that play a role in explaining D meson lifetime differences and other controversial issues, but we are only following a consistent prescription of the model. Indeed, in the BSW model the coefficients a_1 and a_2 are only phenomenological

parameters and do not exactly correspond to the Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ obtained in perturbative QCD and associated with the quark amplitudes. The other term, which is the only relevant term here, corresponds to $\langle \sigma(p') | J_A^{\mu(\bar{d}c)} | D^+(p) \rangle \langle \pi^+(q) | J_{A,\mu}^{\mu(\bar{u}d)} | 0 \rangle$. After this factorization, one needs the matrix element of the current (notice that only the axial currents J_A contribute, because of the parity of the hadrons involved):

$$\langle \sigma(p') | J_A^{\mu(\bar{d}c)} | D^+(p) \rangle = F_1^{(D\sigma)}(q^2) \left((p + p')^\mu - \frac{(m_D^2 - m_\sigma^2)}{q^2} q^\mu \right) + F_0^{(D\sigma)}(q^2) \frac{(m_D^2 - m_\sigma^2)}{q^2} q^\mu, \quad (13)$$

with $F_1(0) = F_0(0)$ in order to avoid a spurious singularity at $q^2 = 0$. The two form factors F_1 and F_0 correspond to transverse and longitudinal components of the current, respectively. If these form factors are dominated by poles, those of F_1 should be at axial vector meson masses and those of F_0 at pseudoscalar meson masses, all of $c\bar{d}$ flavor content. However, only F_0 enters in our case, because $\langle \pi^+(q) | J_A^\mu | 0 \rangle = i f_\pi q^\mu$ is purely longitudinal.

The factor of a_2 in the matrix element $\langle \sigma \pi^+ | H_{eff} | D^+ \rangle$ vanishes, because it involves $\langle \sigma | J^\mu | 0 \rangle$ which is identically zero due to conservation of the vector current. Therefore, from all of the above, the D - σ - π coupling is:

$$\langle \sigma \pi^+ | H_{eff} | D^+ \rangle \equiv g_{D\sigma\pi} = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} a_1 F_0^{(D\sigma)}(m_\pi^2) \times (m_D^2 - m_\sigma^2) f_\pi. \quad (14)$$

Using $V_{cd}^* V_{ud} = 0.21$, the model value $a_1 = 1.10 \pm 0.05$ fitted for D decays [16] and our value for the coupling $g_{D\sigma\pi}$ in Eq. 4, we get an estimate for the axial form factor

$$F_0^{(D\sigma)}(m_\pi^2) = 0.79 \pm 0.15. \quad (15)$$

We can safely extrapolate from $q^2 = m_\pi^2$ to $q^2 = 0$, since we do not expect poles at light masses for a charmed current:

$$F_0(0) = F_1(0) \approx 0.8 \pm 0.2, \quad (16)$$

which agrees within errors with the form factor of Ref. [12] for $D \rightarrow \pi$ and is within the range of most of the other D decays (0.6 to 0.8). A remarkable fact of this result is that

the form factors in Ref. [12] are calculated assuming the mesons are bound states of the corresponding valence quarks, while in our case one would hardly treat *a priori* such a short living resonance like the sigma as a bound state, let alone using a wave function for it. As expected, strong phases do not play a role in the $D^+ \rightarrow \sigma\pi^+$ decay since the final state has only one definite value of isospin, $I = 1$. Strong phases in the (3π) final state have already been taken into account by the introduction of resonances in the fitting procedure to the Dalitz plot.

V. CONCLUSION

We have used the new experimental evidence for the σ meson in the non-leptonic $D^+ \rightarrow \pi^+\pi^+\pi^-$ decay to estimate the effective $g_{D\sigma\pi}$ and $g_{\sigma\pi\pi}$ couplings from the data and the resonance shape. We then studied the consequences of the values of these couplings within the accepted theoretical framework and found consistency of the latter with the experimental data.

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[1] See, *e.g.*, I. Hinchliffe and A. V. Manohar, [hep-ph/0004186](#) for a review.

- [2] See J. Gasser, *Nucl. Phys.* **B279**, 65 (1987) and references therein.
- [3] M. Gell-Mann and M. Lévy, *Nuovo Cimento* **16**, 705 (1960); S. Weinberg, *Phys. Rev. Lett.* **18**, 188 (1967).
- [4] V. Elias and M. Scadron, *Phys. Rev. Lett.* **53**, 1129 (1984); R. Kaminski, L. Lesniak and J.P. Maillet, *Phys. Rev.* **D50**, 3145 (1994); N. Törnqvist, *Z. Phys.* **C68**, 647 (1995); N. Törnqvist and M. Roos, *Phys. Rev. Lett.* **76**, 1575 (1996); *ibid Phys. Rev. Lett.* **77**, 2333 (1996) and *Phys. Rev. Lett.* **78**, 1604 (1997); N. Isgur and J. Speth, *Phys. Rev. Lett.* **77**, 2332 (1996); M. Harada *et al.*, *Phys. Rev.* **D54**, 1991 (1996); *ibid Phys. Rev. Lett.* **78**, 1603 (1997). For more recent views on the problem, see M.R. Pennington, preprint [hep-ph/9905241](#), talk given at *Workshop on Hadron Spectroscopy* (WHS 99), Rome, Italy, 8-12 Mar 1999; S. Ishida, preprint [hep-ph/9905260](#), talk given at *Workshop on Hadron Spectroscopy* (WHS 99), Rome, Italy, 8-12 Mar 1999; L. Celenza, H. Huang and C. Shakin, CUNY preprint BBCCNT 00/051/290 (May 2000), submitted to *Phys. Rev. C*.
- [5] See, *e.g.*, N.A. Törnqvist, summary talk at *Conference on the Possible Existence of the Light σ Resonance and its Implications to Hadron Physics*, Yukawa Institute for Theoretical Physics, Kyoto, Japan, 11-14th June 2000, [hep-ph/0008135](#).
- [6] J.E. Augustin *et al.* (DM2 coll.), *Nucl. Phys.* **B320**, 1 (1989); C. Amsler *et al.*, (Crystal Barrel coll.), *Phys. Lett.* **B291**, 347 (1992); *ibid B322*, 431 (1994); **B342**, 433 (1995); **B353**, 571 (1995); F. Antinori *et al.* (WA91 coll.), *Phys. Lett.* **B353**, 589 (1995); S. Ishida *et al.*, *Progr. Theor. Phys.* **95**, 745 (1995); M. Ishida *et al.*, *Progr. Theor. Phys.* **101**, 601 (1999); *ibid*, preprint [hep-ph/9905261](#), to appear in proc. of *Workshop on Hadron Spectroscopy* (WHS99), Rome, Italy, 8-12 Mar 1999.
- [7] E. van Beveren *et al.*, *Z. Phys.* **C30**, 615 (1986); J. Oller *et al.*, *Nucl. Phys.* **A620**, 438 (1997); *ibid Phys. Rev.* **D59**, 074001 (1999) and **D60**, 074023 (1999); R. Kaminski *et al.*, *Eur. Phys. J.* **C9**, 141 (1999); M. Scadron, *Eur. Phys. J.* **C6**, 141 (1999).

- [8] T. Morozumi, C.S. Lim and A.I. Sanda, *Phys. Rev. Lett.* **65**, 404 (1990).
- [9] J. Miranda, talk presented at the *XX Meeting of the Brazilian Physical Society- Session Particles and Fields*, October 25 - 28, 1999; A.J. Slaughter, talk presented at the *XXXVth Rencontres de Moriond*, March 18 - 25, 2000; E791 Collaboration, E.M. Aitala *et al.*, [hep-ex/0007028](#), submitted to Physical Review Letters.
- [10] A.D. Polosa, N. A. Törnqvist, M.D. Scadron and V. Elias, preprint [hep-ph/0005106](#).
- [11] R. Gatto, G. Nardulli, A.D. Polosa and N. A. Törnqvist, *Phys. Lett.* **B494**, 168 (2000).
- [12] M. Bauer, B. Stech and M. Wirbel, *Z. Phys.* **C34**, 103 (1987).
- [13] See, *e.g.*, J.L. Rosner, *Phys. Rev.* **D60**, 114026 (1999).
- [14] C. Caso *et al.* (Particle Data Group), *Eur. Phys. J. C* **3**, 1 (1998).
- [15] M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley Publ. Co., 1995), page 349.
- [16] M. Neubert and B. Stech, in *Heavy Flavours II*, edited by A.J. Buras and M. Lindner (World Scientific, 1998), page 294.